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Effects of Final-State Interactions in Hadronic B Decays

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Final-state rescattering effects on the hadronic B decays and their impact on direct and mixing-induced CP asymmetries are examined. Implications for some phenomenologies are briefly discussed.

1. Introduction

Although the importance of final-state interactions (FSIs) has long been recognized in hadronic charm decays, the general folklore for hadronic B decays is that FSIs are expected to play only a minor role there due to the large energy release in the energetic B decay. However, there are growing hints at some possible soft final-state rescattering effects in B decays. The measurement of the color-suppressed modes $B^0 \to D^0\pi^0$, when combined with the color-allowed $B \to D\pi$ decays, indicates non-vanishing relative strong phases among various $B \to D\pi$ decay amplitudes. Denoting $\mathcal{T}$ and $\mathcal{C}$ as the color-allowed tree amplitude and color-suppressed W-emission amplitude, respectively, it is found that $\mathcal{C}/\mathcal{T} \sim 0.45 \exp(\pm i60°)$ (see e.g. [1]), showing a non-trivial relative strong phase between $\mathcal{C}$ and $\mathcal{T}$ amplitudes. This is the first evidence of large strong phases in charmful hadronic B decays. Hence, it is natural to expect that sizable strong phases can also manifest in charmless B decays. Since the perturbative strong phases are small in the conventional factorization approach, it is likely that FSIs are responsible for the soft strong phases.

One of the clear indications of large strong phases in charmless hadronic B decays arises from the recent measurements of direct CP violation. The first evidence of direct CP violation was reported by Belle [2] in $B^0 \to \pi^+\pi^-$ even before the $K^\pm\pi^\mp$ modes, but it has not been confirmed by BaBar.[3] A first confirmed observation of direct CP asymmetry was established last year in the charmless B decays $\overline{B}^0(B^0) \to K^\mp\pi^\pm$ by both BaBar [4] and Belle. [5] Also the combined BaBar and Belle measurements of $\overline{B}^0 \to \rho^\pm\pi^\mp$ imply a 3.6σ direct CP asymmetry in the $\rho^+\pi^-$ mode. [6]

Table 1 shows the comparison of the model predictions of direct CP asymmetries with the world averages of experimental results. [6] The agreement of pQCD results [7] with experiment for $K^-\pi^+$ and $\pi^+\pi^-$ is impressive, recalling that pQCD predictions were made years before experiment (for updated pQCD predictions, see

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[8]). In contrast, QCDF predictions [9,10] appear to be lousy as the predicted signs for all the three modes $K^-\pi^+, \rho^+\pi^-, \pi^+\pi^-$ are wrong. This discrepancy has often led to the claims in the literature that QCDF fails to describe direct CP asymmetries. However, this is not the case. As shown in Table I, there exist several theoretical uncertainties in QCDF predictions especially those arising from power corrections dictated by the last entry of theoretical errors. Power corrections always involve endpoint divergences. For example, the $1/m_b$ annihilation amplitude has endpoint divergences even at twist-2 level and the hard spectator scattering diagram at twist-3 order is power suppressed and posses soft and collinear divergences arising from the soft spectator quark. Since the treatment of endpoint divergences is model dependent, subleading power corrections generally can be studied only in a phenomenological way. While the endpoint divergence is regulated in the pQCD approach by introducing the parton's transverse momentum, [7] it is parameterized in QCD factorization as

$$X_A \equiv \int_0^1 \frac{dy}{y} = \ln\frac{m_B}{\Lambda_h}(1 + \rho_A e^{i\phi_A}), \tag{1}$$

with $\rho_A \leq 1$ and Λ_h being a typical scale of order 500 MeV.

Table 1.   Comparison of pQCD and QCD factorization (QCDF) predictions of direct CP asymmetries (in %) with experiment. Also shown are the QCDF results in scenario 4 denoted by QCDF(S4)[10] and the FSI modifications to short-distance (SD) predictions.[11] The pQCD results for $\rho\pi$ modes are taken from [12].

Mode	Expt.	pQCD	QCDF	QCDF(S4)	SD+FSI
$\overline{B}^0 \to K^-\pi^+$	-11 ± 2	-17 ± 5	$4.5^{+1.1+2.2+0.5+8.7}_{-1.1-2.5-0.6-9.5}$	-4.1	-15^{+3}_{-1}
$\overline{B}^0 \to \rho^+\pi^-$	-47^{+13}_{-14}	$-7.1^{+0.1}_{-0.2}$	$0.6^{+0.2+1.3+0.1+11.5}_{-0.1-1.6-0.1-11.7}$	-12.9	-43 ± 11
$\overline{B}^0 \to \pi^+\pi^-$	37 ± 10	23 ± 7	$-6.5^{+2.1+3.0+0.1+13.2}_{-2.1-2.8-0.3-12.8}$	10.3	$-$
$\overline{B}^0 \to \pi^0\pi^0$	28 ± 39	30 ± 10	$45.1^{+18.4+15.1+4.3+46.5}_{-12.8-13.8-14.1-61.6}$	-19	-30^{+1}_{-4}
$\overline{B}^0 \to \rho^-\pi^+$	-15 ± 9	12 ± 2	$-1.5^{+0.4+1.2+0.2+8.5}_{-0.4-1.3-0.3-8.4}$	3.9	-24 ± 6

Because of the large uncertainties in power corrections, one may wonder if it is possible to accommodate the experimental measurements of direct CP violation in QCDF in certain parameter space. This is indeed the case. Just like the pQCD approach where the annihilation topology plays an essential role for producing sizable strong phases and for explaining the penguin-dominated VP modes, Beneke and Neubert [10] chose a favorable scenario (denoted as S4) to accommodate the observed penguin-dominated $B \to PV$ decays and the measured sign of direct CP asymmetry in $\overline{B}^0 \to K^-\pi^+$ by choosing $\rho_A = 1$, $\phi_A = -55°$ for PP, $\phi_A = -20°$ for PV and $\phi_A = -70°$ for VP modes. The sign of ϕ_A is chosen so that the direct CP violation $A_{K^-\pi^+}$ agrees with the data. It is clear from Table I that the signs of $A_{\pi^+\pi^-}$ and $A_{\rho^+\pi^-}$ are correctly reproduced in QCD(S4). In short, one needs large strong phases to explain the observed direct CP violation in B decays.

For given ρ_A and ϕ_A, one can claim that QCDF still makes predictions. However, the origin of these phases and large annihilation magnitude is unknown. Moreover,

the annihilation topologies do not help enhance the $\pi^0\pi^0$ and $\rho^0\pi^0$ modes. Note that neither pQCD nor QCDF can explain the large direct CP asymmetry observed in $\overline{B}^0 \to \rho^+\pi^-$. Hence, one would wish to have an explanation of the data without invoking weak annihilation. Therefore, it is of great importance to study final-state rescattering effects on decay rates and CP violation.

Besides the above-mentioned CP violation, there exist several other hints at large FSI effects in the B sector. [11] For example, the measured branching ratio [6] $\mathcal{B}(B^0 \to \pi^0\pi^0) = (1.5 \pm 0.3) \times 10^{-6}$ cannot be explained by either QCDF or pQCD and this may call for a possible rescattering effect to induce $\pi^0\pi^0$. The QCDF predictions for penguin-dominated modes such as $B \to K^*\pi$, $K\rho$, $K\phi$, $K^*\phi$ are consistently lower than the data by a factor of 2 to 3. [10] This large discrepancy between theory and experiment indicates the importance of subleading power corrections such as the annihilation topology and/or FSI effects.

Our goal is to study FSI effects on branching ratios and direct CP asymmetries in B decays. Long distance (LD) rescattering effects can be included in any SD approach but it requires modelling of the $1/m_b$ power corrections. In [11] we present a specific model for FSI (to be described in the next section) to predict (rather than accommodate) the sign and magnitude of direct CP violation.

2. Final State Interactions in Hadronic B decays

In QCDF there are two hard strong phases: one from the absorptive part of the penguin graph in $b \to s(d)$ transitions [13] and the other from the vertex corrections. However, these perturbative strong phases do not lead to the correct sign of direct CP asymmetries observed in $K^-\pi^+$, $\rho^+\pi^-$ and $\pi^+\pi^-$ modes. Therefore, one has to consider the nonpertrubative strong phases induced from power suppressed contributions such as FSIs. The idea is that if the intermediate states are CKM more favored than the final state, e.g. charm intermediate states in charmless B decays, then the absorptive part of the final-state rescattering amplitude can easily give rise to large strong phases and make significant contributions to the rates.

Based on the Regge approach, Donoghue *et al.* [14] have reached the interesting conclusion that FSIs do not disappear even in the heavy quark limit and soft FSI phases are dominated by inelastic scattering, contrary to the common wisdom. However, it was later pointed out by Beneke *et al.* [9] within the framework of QCD factorization that the above conclusion holds only for individual rescattering amplitudes. When summing over all possible intermediate states, there exist systematic cancellations in the heavy quark limit so that the strong phases must vanish in the limit of $m_b \to \infty$. Consequently, the FSI phase is generally of order $\mathcal{O}(\alpha_s, \Lambda_{\rm QCD}/m_b)$. In reality, because the b quark mass is not very large and far from being infinity, the aforementioned cancellation may not occur or may not be very effective for the finite B mass. Hence, the strong phase arising from power corrections can be in principle very sizable.

At the quark level, final-state rescattering can occur through quark exchange

and quark annihilation. In practice, it is extremely difficult to calculate the FSI effects, but it may become amenable at the hadron level where FSIs manifest as the rescattering processes with s-channel resonances and one particle exchange in the t-channel. In contrast to D decays, the s-channel resonant FSIs in B decays is expected to be suppressed relative to the rescattering effect arising from quark exchange owing to the lack of the existence of resonances at energies close to the B meson mass. Therefore, we will model FSIs as rescattering processes of some intermediate two-body states with one particle exchange in the t-channel and compute the absorptive part via the optical theorem. [11]

The approach of modelling FSIs as soft rescattering processes of intermediate two-body states has been criticized on several grounds. [9] For example, there are many more intermediate multi-body channels in B decays and systematic cancellations among them are predicted to occur in the heavy quark limit. This effect of cancellation will be missed if only a few intermediate states are taken into account. As mentioned before, the cancellation may not occur or may not be very effective as the B meson is not infinitely heavy. Hence, we may assume that two-body $\rightleftharpoons$ n-body rescatterings are negligible either justified from the $1/N_c$ argument or suppressed by large cancellations. Indeed, it has been even conjectured that the absorptive part of long-distance rescattering is dominated by two-body intermediate states, while the dispersive part is governed by multi-body states. [15] At any rate, we view our treatment of the two-body hadronic model for FSIs as a working tool. We work out the consequences of this tool to see if it is empirically working. If it turns out to be successful, then it will imply the possible dominance of intermediate two-body contributions.

The calculations of hadronic diagrams for FSIs involve many theoretical uncertainties. Since the particle exchanged in the t channel is off shell and since final state particles are hard, form factors or cutoffs must be introduced to the strong vertices to render the calculation meaningful in perturbation theory. We shall parametrize the off-shell effect of the exchanged particle as

$$F(t, m) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 - t} \right)^n, \tag{2}$$

normalized to unity at $t = m^2$ with m being the mass of the exchanged particle. The monopole behavior of the form factor (i.e. $n = 1$) is preferred as it is consistent with the QCD sum rule expectation. [16] For the cutoff Λ, it should be not far from the physical mass of the exchanged particle. To be specific, we write $\Lambda = m_{\mathrm{exc}} + r\Lambda_{\mathrm{QCD}}$ where the parameter r is expected to be of order unity and it depends not only on the exchanged particle but also on the external particles involved in the strong-interaction vertex. As we do not have first-principles calculations for form factors, we shall use the measured decay rates to fix the unknown cutoff parameters and then use them to predict direct CP violation.

As mentioned in the Introduction, final-state rescattering effects can be implemented in any SD approach. For our purpose, we shall choose QCD factorization as

the short-distance framework to start with. Moreover, we should set $\rho_{A,H}$ to zero in order to avoid the double counting problem.

2.1   Penguin dominated modes

Penguin dominated modes such as $B \to K\pi,\ K^*\pi,\ K\rho,\ \phi K^{(*)}$ receive sizable contributions from rescattering of charm intermediate states (i.e. the so-called long-distance charming penguins). For example, the branching ratios of $B \to \phi K$ and ϕK^* can be enhanced from $\sim 5 \times 10^{-6}$ predicted by QCDF to the level of 1×10^{-5} by FSIs via rescattering of charm intermediate states. [11] The decay $\overline{B}^0 \to K^{*-}\pi^+$ predicted at the level 3.8×10^{-9} by QCDF is enhanced by final-state rescattering to the order of 10×10^{-6}, to be compared with $(12.7^{+1.8}_{-1.7}) \times 10^{-6}$ experimentally. [6]

2.2   Tree dominated modes

The color-suppressed $\rho^0\pi^0$ mode is slightly enhanced by rescattering effects to the order of 1.3×10^{-6}, which is consistent with the weighted average $(1.9 \pm 0.6) \times 10^{-6}$ of the experimental values, $(1.4 \pm 0.7) \times 10^{-6}$ by BaBar [17] and $(3.12^{+0.88+0.60}_{-0.82-0.76}) \times 10^{-6}$ by Belle. [18] Note that the branching ratio of $\rho^0\pi^0$ is predicted to be of order 0.2×10^{-6} in the pQCD approach, [19] which is too small compared to experiment as the annihilation contribution does not help enhance its rate.

2.3   Direct *CP* asymmetries

The strong phases in charmless B decays are governed by final-state rescattering. We see from the last column of Table 1 that direct CP-violating partial rate asymmetries in $K^-\pi^+$ and $\rho^+\pi^-$ are significantly affected by final-state rescattering and their signs are different from that predicted by the short-distance QCDF approach. Direct CP violation in $\pi^+\pi^-$ cannot be predicted in this approach as charming penguins are not adequate to explain the $\pi\pi$ data: the predicted $\pi^+\pi^-$ ($\sim 9 \times 10^{-6}$) is too large whereas $\pi^0\pi^0$ ($\sim 0.4 \times 10^{-6}$) is too small. This means that a dispersive long-distance contribution is needed to interfere destructively with $\pi^+\pi^-$ so that $\pi^+\pi^-$ will be suppressed while $\pi^0\pi^0$ will get enhanced. One needs the observed $\pi\pi$ rates and $A_{\pi^+\pi^-}$ to fix this new LD contribution (see [11] for details).

Direct CP asymmetries in $\overline{B}^0 \to \pi^0\pi^0, \rho^-\pi^+$ decays are also shown in Table 1 where we see that the predictions of pQCD and the SD approach supplemented with FSIs are opposite in sign. It will be interesting to measure direct CP violation in these two decays to test different models.

3.   Mixing-induced *CP* violation

Considerable activity in search of possible New Physics beyond the Standard Model has recently been devoted to the measurements of time-dependent CP asymmetries

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in neutral B meson decays into final CP eigenstates defined by

$$\frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to f)}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to f)} = S_f \sin(\Delta mt) + A_f \cos(\Delta mt), \tag{3}$$

where Δm is the mass difference of the two neutral B eigenstates, S_f monitors mixing-induced CP asymmetry and A_f measures direct CP violation. The time-dependent CP asymmetries in the $b \to sq\bar{q}$ penguin-induced two-body decays such as $B^0 \to (\phi, \omega, \pi^0, \eta', f_0)K_S$ and three-body decays e.g. $B^0 \to K^+K^-K_S^0, K_S^0 K_S^0 K_S^0$ measured by BaBar [20] and Belle [21] show some indications of sizable deviations from the expectation of the SM where CP asymmetry in all above-mentioned modes should be equal to $\sin 2\beta$ inferred from the $B^0 \to J/\psi K$ decay with a small deviation *at most* $\mathcal{O}(0.1)$. [22] In order to detect the signal of New Physics unambiguously in the penguin $b \to s$ modes, it is of great importance to examine how much of the deviation of S_f from $S_{J/\psi K}$ is allowed in the SM. Based on the framework of QCD factorization, the mixing-induced CP violation parameter S_f in the seven 2-body modes $(\phi, \omega, \rho^0, \eta', \eta, \pi^0, f_0)K_S$ has recently been quantitatively studied in [23] and [24]. It is found that the sign of $\Delta S_f \equiv -\eta_f S_f - S_{J/\psi K_S}$ (η_f being the CP eigenvalue of the final state f) at short distances is positive except for the channel $\rho^0 K_S$.

In all previous studies and estimates of ΔS_f, effects of FSI were not taken into account. In view of the striking observation of large direct CP violation in $B^0 \to K^{\pm}\pi^{\mp}$, it is clear that final-state phases in two-body B decays may not be small. It is therefore important to understand their effects on ΔS_f. It is found [23] that the long-distance effects on S_f are generally negligible except for the ωK_S and $\rho^0 K_S$ modes where S_f is lowered by around 15% for the former and enhanced by the same percentage for the latter and $\Delta S_{\omega K_S, \rho^0 K_S}^{SD+LD}$ become consistent with zero within errors. Moreover, the central values of ΔS_f become positive for all the modes under consideration, but they tend to be rather small compared to the theoretical uncertainties involved so that it is difficult to make reliable statements on the sign at present. [23]

Table 2.   Direct CP asymmetry parameter A_f and the mixing-induced CP parameter ΔS_f^{SD+LD} for various modes. The first and second theoretical errors correspond to the SD and LD ones, respectively (see [23] for details).

State	ΔS_f			$A_f(\%)$		
	SD	SD+LD	Expt	SD	SD+LD	Expt
ϕK_S	$0.02^{+0.00}_{-0.04}$	$0.03^{+0.01+0.01}_{-0.04-0.01}$	-0.38 ± 0.20	$1.4^{+0.3}_{-0.5}$	$-2.6^{+0.8+0.0}_{-1.0-0.4}$	4 ± 17
ωK_S	$0.12^{+0.05}_{-0.06}$	$0.01^{+0.02+0.02}_{-0.04-0.01}$	$-0.17^{+0.30}_{-0.32}$	$-7.3^{+3.5}_{-2.6}$	$-13.2^{+3.9+1.4}_{-2.8-1.4}$	48 ± 25
$\rho^0 K_S$	$-0.09^{+0.03}_{-0.07}$	$0.04^{+0.09+0.08}_{-0.10-0.11}$		$9.0^{+2.2}_{-4.6}$	$46.6^{+12.9+3.9}_{-13.7-2.6}$	
$\eta' K_S$	$0.01^{+0.00}_{-0.04}$	$0.00^{+0.00+0.00}_{-0.04-0.00}$	-0.30 ± 0.11	$1.8^{+0.4}_{-0.4}$	$2.1^{+0.5+0.1}_{-0.2-0.1}$	4 ± 8
ηK_S	$0.07^{+0.02}_{-0.04}$	$0.07^{+0.02+0.00}_{-0.05-0.00}$		$-6.1^{+5.1}_{-2.0}$	$-3.7^{+4.4+1.4}_{-1.8-2.4}$	
$\pi^0 K_S$	$0.06^{+0.02}_{-0.04}$	$0.04^{+0.02+0.01}_{-0.03-0.01}$	$-0.39^{+0.27}_{-0.29}$	$-3.4^{+2.1}_{-1.1}$	$3.7^{+3.1+1.0}_{-1.7-0.4}$	-8 ± 14

Recently, we have also studied the decay rates and time-dependent CP asymmetries in the three-body decays $B^0 \to K^+K^-K_{S(L)}$ and $K_S K_S K_{S(L)}$ within the

framework of factorization. [25] Owing to the presence of color-allowed tree contributions in $B^0 \to K^+K^-K_{S(L)}$, this penguin-dominated mode is subject to a significant tree pollution and the deviation of the mixing-induced CP asymmetry from that measured in $B \to J/\psi K_S$, namely, $\Delta \sin 2\beta_{K^+K^-K_{S(L)}} \equiv \sin 2\beta_{K^+K^-K_{S(L)}} - \sin 2\beta_{J/\psi K_S}$, can be as large as $\mathcal{O}(0.10)$. The deviation $\Delta \sin 2\beta_{K^+K^-K_{S(L)}}$ arises mainly from the large $m_{K^+K^-}$ region.

4. Polarization Anomaly in $B \to \phi K^*$

For $B \to V_1 V_2$ decays with V being a light vector meson, it is expected that they are dominated by longitudinal polarization states and respect the scaling law: $1 - f_L = \mathcal{O}(m_V^2/m_B^2)$. However, a low value of the longitudinal fraction $f_L \approx 50\%$ and sizable perpendicular polarization $f_\perp \approx 20\%$ in ϕK^* decays were observed by both BaBar [26] and Belle [27] (see Table 3). The polarization anomaly for f_L poses an interesting challenge for any theoretical interpretation.

Table 3.   Experimental data for CP averaged branching ratios (in units of 10^{-6}) and polarization fractions for $B \to \phi K^*$ and ρK^*. [27,28]

Mode	BaBar	Belle	Average
$f_L(\phi K^{*0})$	$0.52 \pm 0.05 \pm 0.02$	$0.45 \pm 0.05 \pm 0.02$	0.48 ± 0.04
$f_\perp(\phi K^{*0})$	$0.22 \pm 0.05 \pm 0.02$	$0.31^{+0.06}_{-0.05} \pm 0.02$	0.26 ± 0.04
$f_L(\phi K^{*+})$	$0.46 \pm 0.12 \pm 0.03$	$0.52 \pm 0.08 \pm 0.03$	0.50 ± 0.07
$f_\perp(\phi K^{*+})$		$0.19^{+0.08}_{-0.02}$	0.19 ± 0.08
$f_L(\rho^0 K^{*+})$	$0.96^{+0.04}_{-0.15} \pm 0.04$		$0.96^{+0.06}_{-0.15}$
$f_L(\rho^+ K^{*0})$	$0.79 \pm 0.08 \pm 0.04$	$0.43 \pm 0.11^{+0.05}_{-0.02}$	0.66 ± 0.07

Working in the context of QCD factorization, it has been argued that the lower value of the longitudinal polarization fraction and the large transverse rate can be "accommodated" by the $(S-P)(S+P)$ penguin-induced annihilation contributions. [28] This is so because the transverse polarization amplitude induced from the above annihilation topologies is of the same $1/m_b$ order as the longitudinal one. For other alternative solutions to the ϕK^* anomaly, see [29].

Contrary to the SD approach, it is considerably easy to obtain a large transverse polarization via final state rescattering. To illustrate the idea, consider the long-distance rescattering processes from the intermediate states $D^{(*)}D_s^{(*)}$ [11,30]. It is easy to find out the polarization states of $D^*D_s^*$ in $B \to D^*D_s^*$ to be $f_L \approx 0.51$, $f_\parallel \approx 0.41$ and $f_\perp \approx 0.08$. Hence, the large transverse polarization induced from $B \to D^*D_s^*$ can be propagated to ϕK^* via FSI rescattering. What about $f_\perp$? It is obvious that the perpendicular polarization induced from $D^{(*)}D_s^{(*)}$ through rescattering is too small to account for experiment. Indeed, $f_\perp$ vanishes in $m_c/m_b \to 0$ limit. Nevertheless, rescattering from $B \to D^*D_s$ or $B \to DD_s^*$ have unique contributions to the $A_\perp$ amplitude.

However, we found no sizable perpendicular polarization owing mainly to the large cancellations occurring in the processes $B \to D_s^*D \to \phi K^*$ and $B \to D_s D^* \to$

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ϕK^* and this can be understood as a consequence of *CP* and SU(3) symmetry. [11] Our result is thus drastically different from a recent similar study in [30]. In short, it is "trivial" to get a large ϕK^* transverse polarization via LD rescattering, but it takes some efforts e.g. the *p*-wave charm intermediate states [11] to circumvent the aforementioned CPS constraint and achieve sizable perpendicular polarization.

As for the transverse polarization in $B \to \rho K^*$ decays, both final-state rescattering and large annihilation scenarios lead to $f_L(\rho K^*) \sim 60\%$. However, none of the existing models can explain the observed disparity between $f_L(\rho^+ K^{*0})$ and $f_L(\rho^0 K^{*+})$. This should be clarified both experimentally and theoretically.

5. Acknowledgments

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